Simple Variants of Non-cooperative Polymorphic P Systems

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Outline

- Polymorphic P systems
  - The idea and the model
  - A few basic properties
- Polymorphic P systems with non-cooperative rules and no ingredients
- Non-cooperative polymorphic P systems with limited depth
- Non-cooperative polymorphic P systems with "finitely representable" regions

Outline – Overview of earlier results

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Polymorphic P systems - The idea

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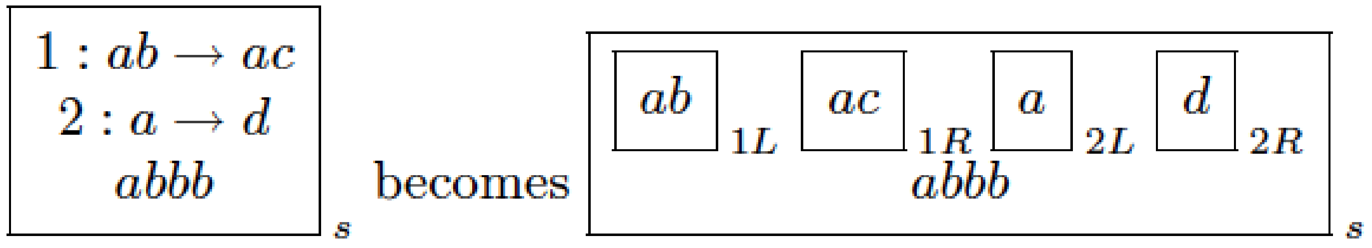
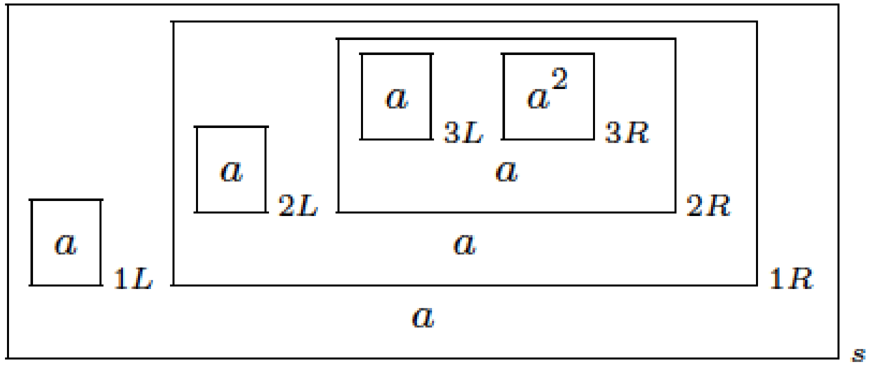
- Sergiu Ivanov: Polymorphic P Systems with Non-cooperative Rules and No Ingredients.
  In: *CMC 2014*, Vol. 8961 of *LNCS*, pp. 258-273, 2014

Polymorphic P systems - The idea

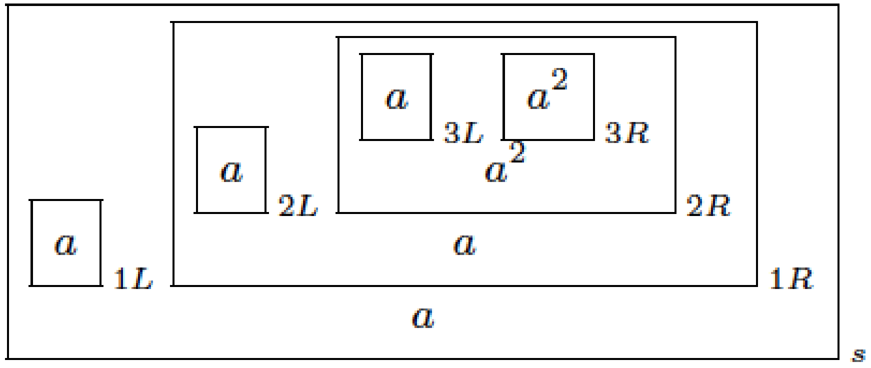
- To manipulate the **rules** during a computation: **represent** them **as data**



For example



$3 : a \to a^2$ in $2R$
$2 : a \to a$ in $1R$
$1 : a \to a$ in s
$\Rightarrow$



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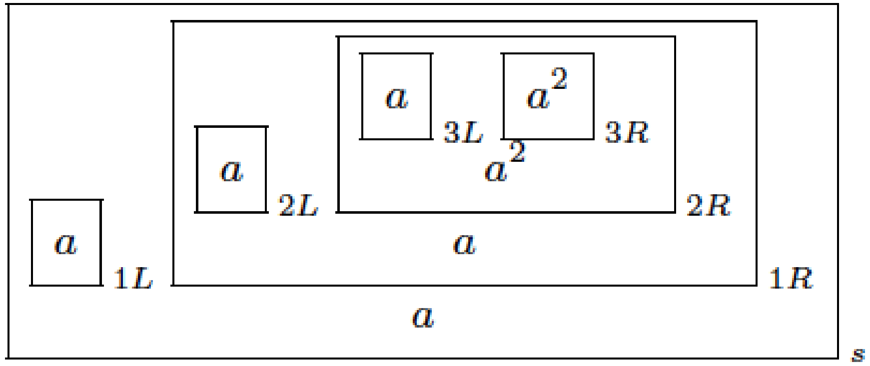

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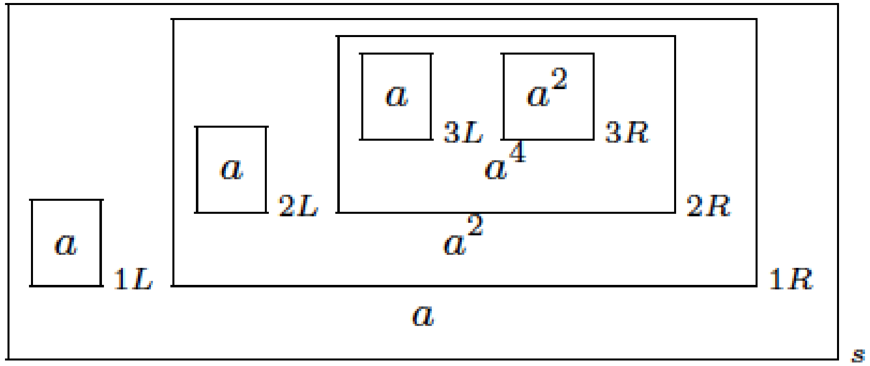
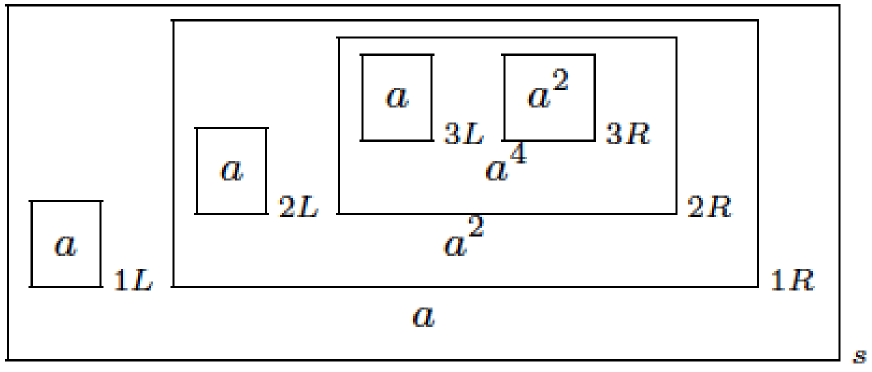


$3 : a \rightarrow a^2$ in $2R$
$2 : a \rightarrow a^4$ in $1R$
$1 : a \rightarrow a^2$ in s
$\Rightarrow$

For example



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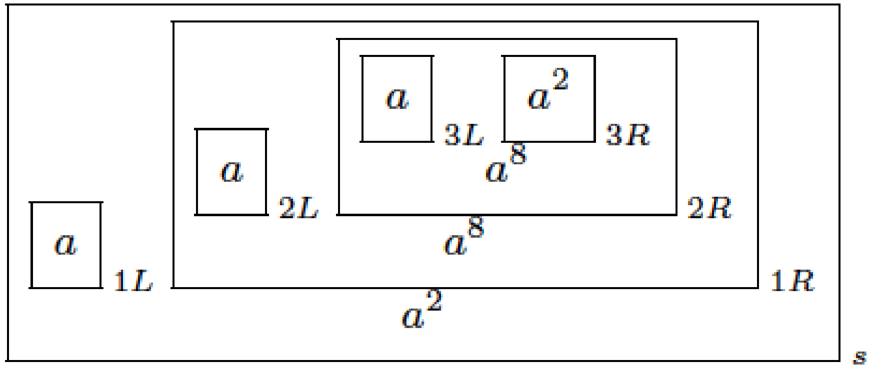

$3 : a \to a^2$ in $2R$
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$\Rightarrow$

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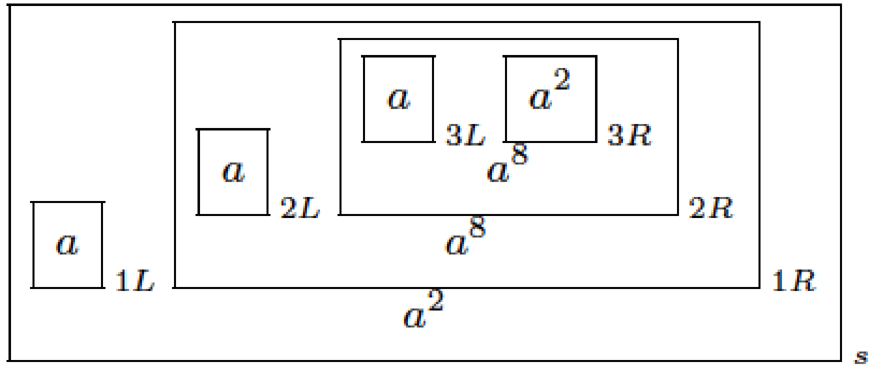
For example



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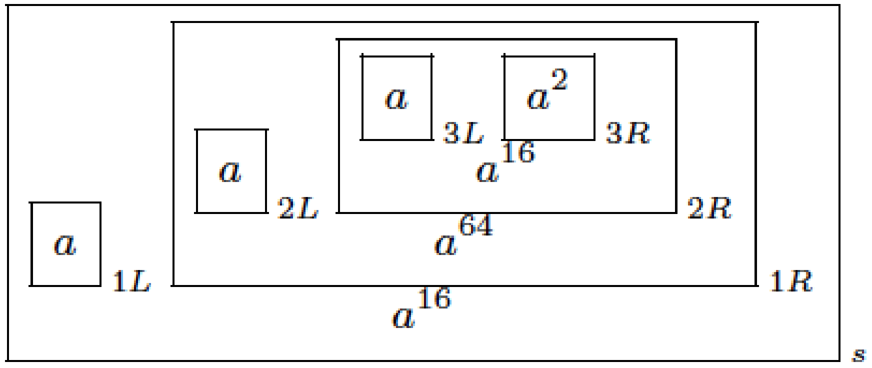

$3 : a \rightarrow a^2$ in $2R$
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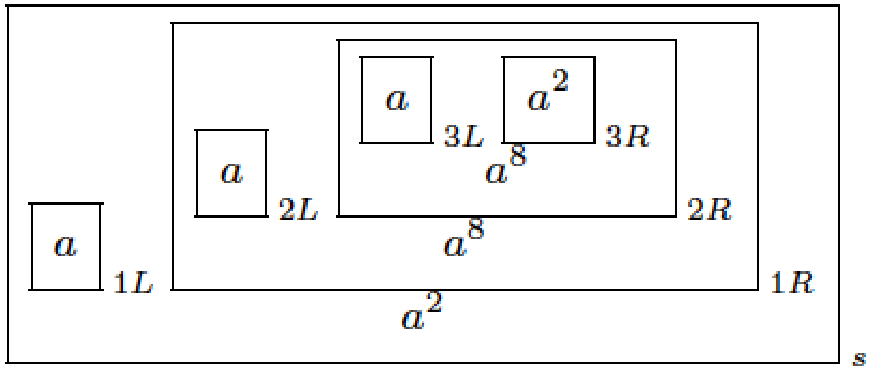
$\Rightarrow \cdots$

For example



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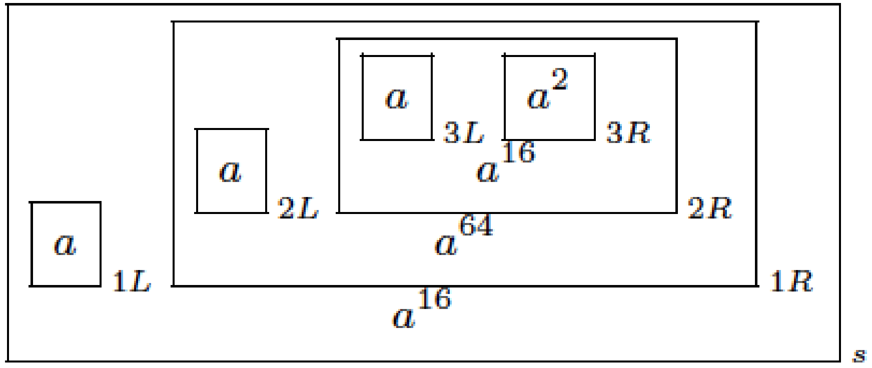

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$\Rightarrow \cdots$

$$(2, 2^n, 2^{n(n-1)/2}, 2^{n(n-1)(n-2)/6})$$

Notation

$$(D)OP_k(polym_{+d}(coo), tar)$$

$$NOP_k(polym_{+d}(coo), tar),\ \ PsOP_k(polym_{+d}(coo), tar),$$

- Numbers, vectors
- The number of regions
- Rule “disabling”
- Cooperating, non-cooperating rules
- Target indicators
- ….

Theorem 2. *There exist*

- *A strongly universal P system from* $OP_{47}(polym_{-d}(coo))$*;*
- *A P system* $\Pi_1 \in DOP_7(polym_{-d}(ncoo))$ *with a superexponential growth;*
- *A P system* $\Pi_2 \in OP_{13}(polym_{-d}(ncoo), tar)$ *such that* $N(\Pi_2) = \{n! \cdot n^k \mid n \geq 1,\ k \geq 0\}$ *and the time complexity of generating* $n! \cdot n^k$ *is* $n + k + 1$*;*
- *A P system* $\Pi_3 \in OP_9(polym_{-d}(coo), tar)$ *such that* $N(\Pi_3) = \{n! \mid n \geq 1\}$ *and the time complexity of generating* $n!$ *is* $n + 1$*;*
- *A P system* $\Pi_4 \in OP_{15}(polym_{-d}(ncoo), tar)$ *such that* $N(\Pi_4) = \{2^{2^n} \mid n \geq 0\}$ *and the time complexity of generating* 2^{2^n} *is* $3n + 2$*;*
- *A P system* $\Pi_5' \in DOP_*(polym_{-d}(coo), tar)$ *such that* $f(\Pi_5) = (n \longrightarrow 2^{2^n})$ *and the time complexity of computing* $n \longrightarrow 2^{2^n}$ *is* $O(n)$*;*
- *A P system* $\Pi_6 \in DOP_*(polym_{-d}(coo), tar)$ *such that* $N_d(\Pi_6) = \{n! \mid n \geq 1\}$ *and the complexity of deciding any number* k*,* $k \leq n!$ *does not exceed* $4n$*.*

Moreover, polymorphic P systems can grow faster than any non-polymorphic P systems, whereas even non-cooperative polymorphic P systems with targets can grow faster than any polymorphic P systems without targets.

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Systems with non-cooperative rules

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What is "no ingredients"?

Variants of non-cooperativity

- **Strong non-cooperative** systems: left membranes contain at most one symbol
- **Weak non-cooperative systems**: all rules which are actually applied have one symbol on their left-hand side

Theorem 2. $NOP_*(polym_{+d}(ncoo_w)) = NOP_*(polym_{+d}(ncoo_s))$.

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Rule disabling doesn't matter

Proposition 1. $NOP_*(polym_{-d}(ncoo)) = NOP_*(polym_{+d}(ncoo))$.

Left polymorphism

- In general, as a consequence of certain lemmas: left membranes with “static rules” are sufficient
- Left polymorphic systems are more powerful than conventional transition P systems, but they cannot generate everything:

Proposition 2. $L_{2^n} = \{2^n \mid n \in \mathbb{N}\} \in NOP_*(lpolym(ncoo))$.

Proposition 3. $L_{n!} = \{n! \mid n \in \mathbb{N}\} \notin NOP_*(lpolym_{+d}(ncoo))$.

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A depth-based hierarchy

Theorem 4. $L_{d+1} = \left\{ 2^{\binom{n}{d-1}} \,\middle|\, n \in \mathbb{N}, n > d \right\} \notin NOP_*^d(polym(ncoo)),\ d > 1.$

Corollary 3. $NOP_*^d(polym(ncoo)) \subsetneq NOP_*^{d+1}(polym(ncoo)).$

As a consequence for left polymorphic systems:

Corollary 4. $NOP_*(lpolym(ncoo)) \subsetneq NOP_*(polym(ncoo)).$

(since for left polymorphic systems, depth 3 is sufficient to reach their maximal power)

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Non-cooperative polymorphic P systems with limited depth

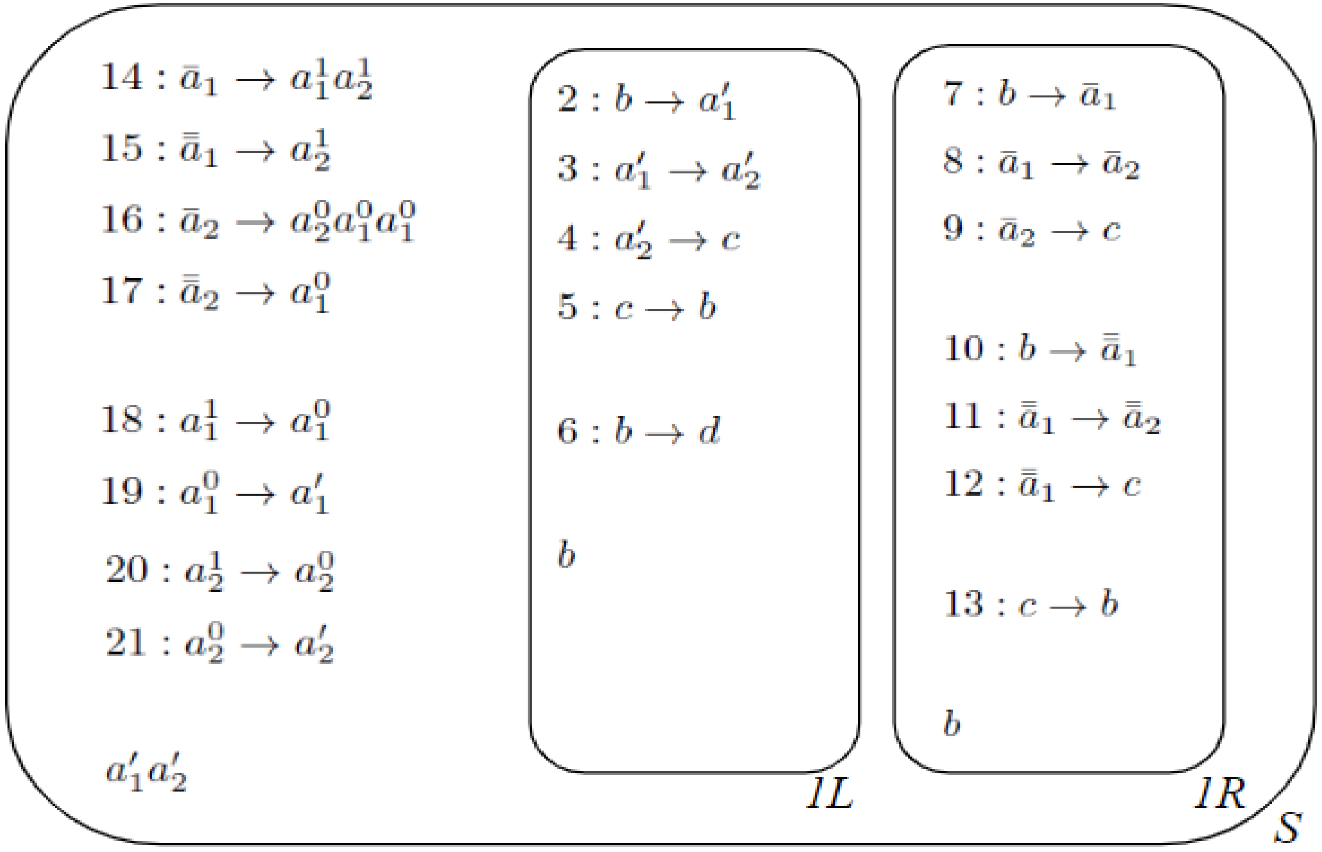
Theorem: $PsET0L \subseteq \mathcal{L}(NOP^3(polym, ncoo))$.

Proof idea – an example

The ET0L system:

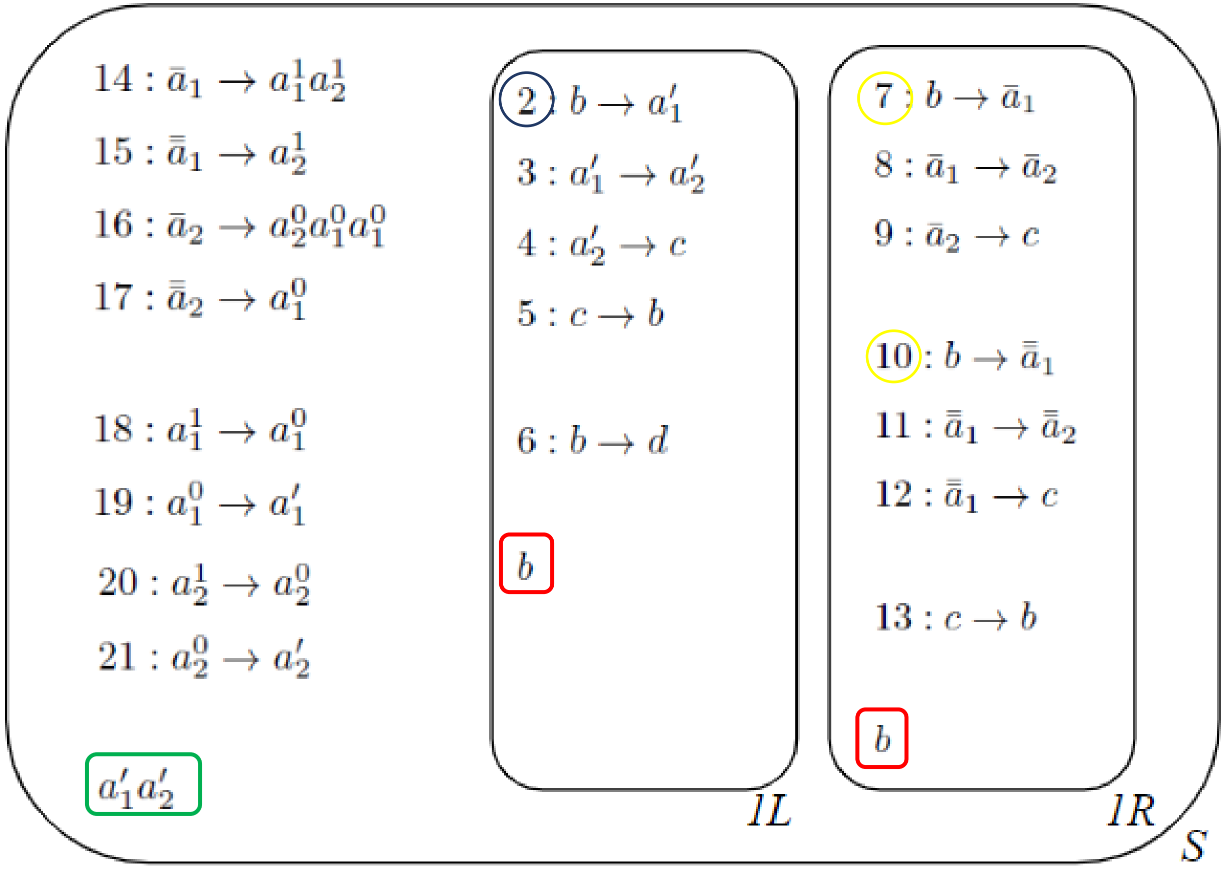
$$
\begin{aligned}
G &= (V, T, U, w)\\
V &= T = \{a_1, a_2\},\\
w &= a_1 a_2,\\
U &= (P_1, P_2),\\
P_1 &= \{a_1 \to a_1 a_2,\ a_2 \to a_2 a_1 a_1\}\\
P_2 &= \{a_1 \to a_2,\ a_2 \to a_1\}.
\end{aligned}
$$

The membrane system:



Proof idea

Step	Rule 1	Contents of the Skin
1.	$b \to b$	$a'_1 a'_2$
2.	$a'_1 \to \bar{a}_1$	$a'_1 a'_2$
3.	$a'_2 \to \bar{a}_2$	$\bar{a}_1 a'_2$
4.	$c \to c$	$a_1^1 a_2^1 \bar{a}_2$
5.	$b \to b$	$a_1^0 a_2^0 a_2^0 a_1^0 a_1^0$
6.	$a'_1 \to \bar{a}_1$ or $a'_1 \to \bar{\bar{a}}_1$	$a'_1 a'_2 a'_2 a'_1 a'_1$



ET0L tables: $P_1 = \{a_1 \to a_1 a_2,\ a_2 \to a_2 a_1 a_1\}$,
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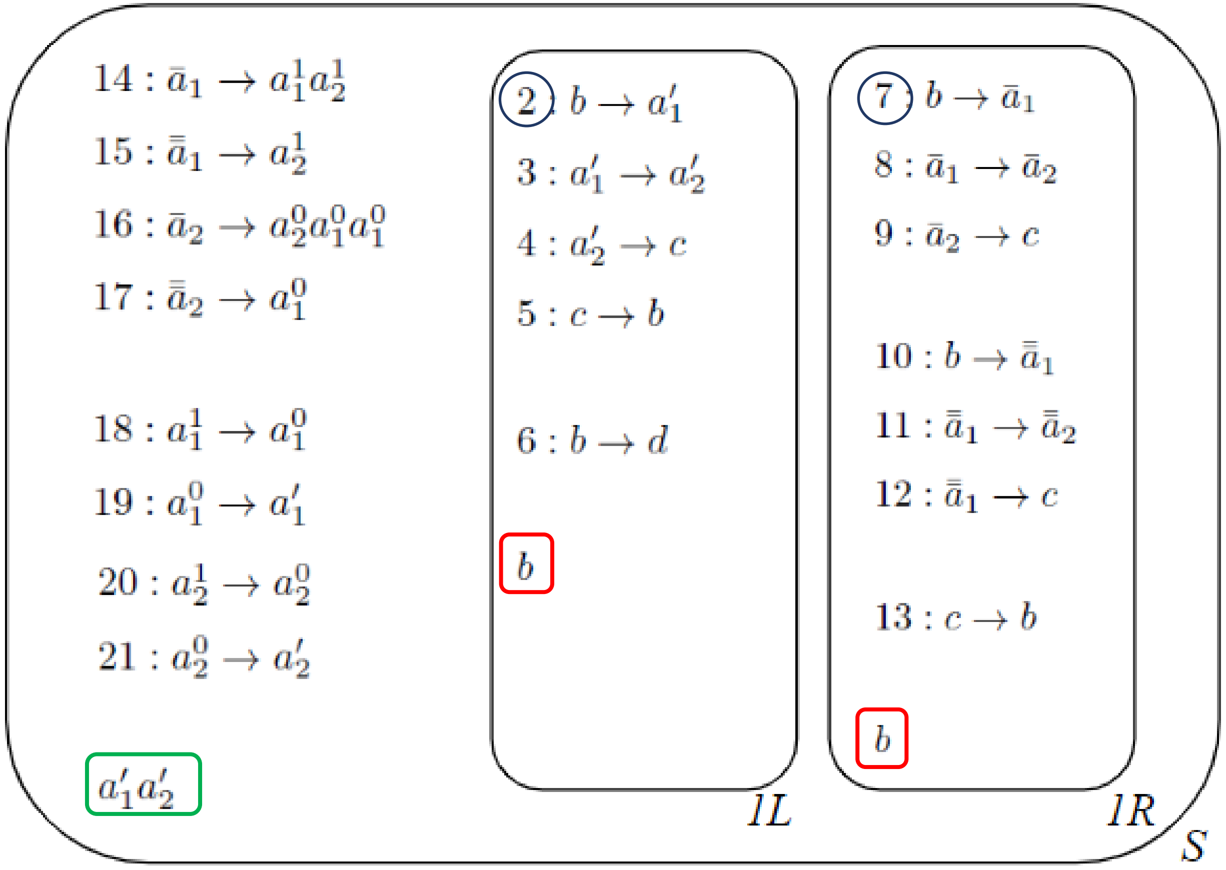

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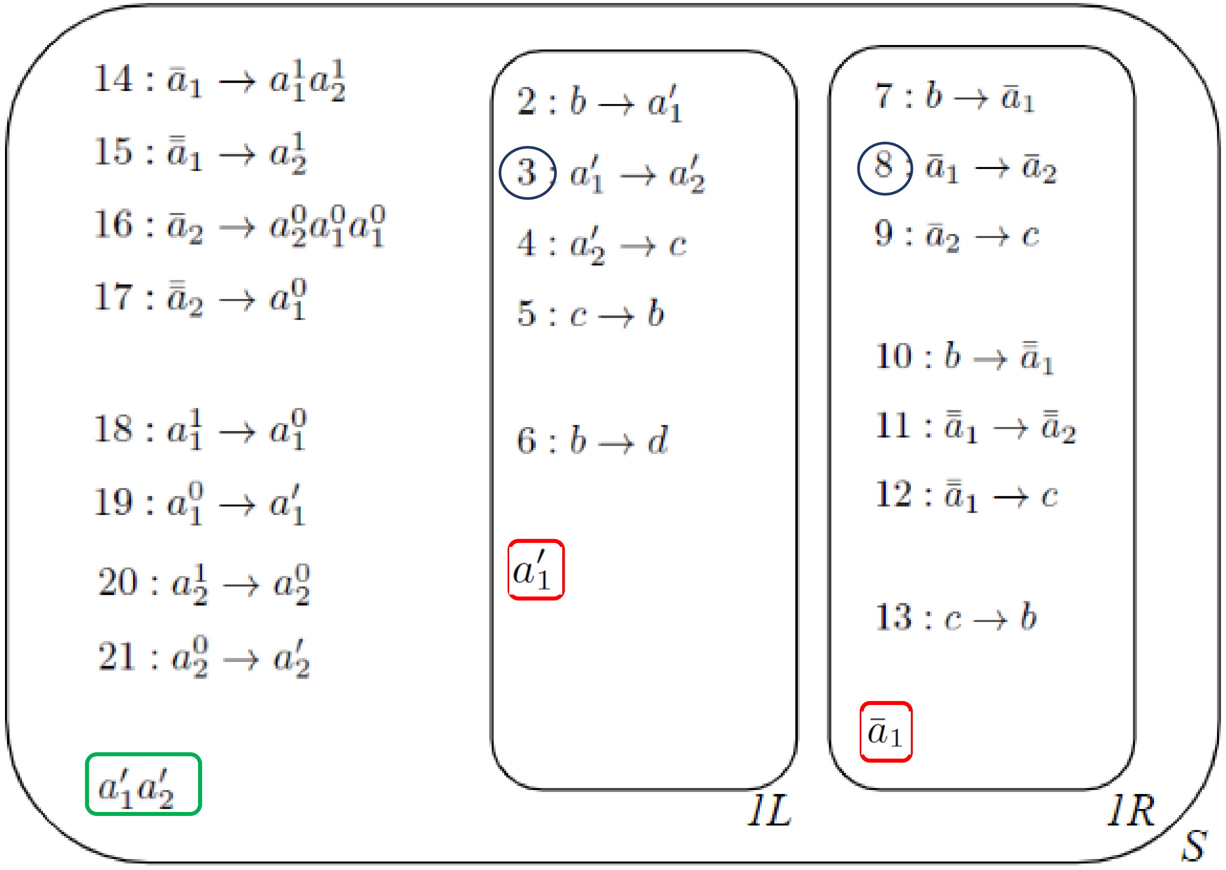

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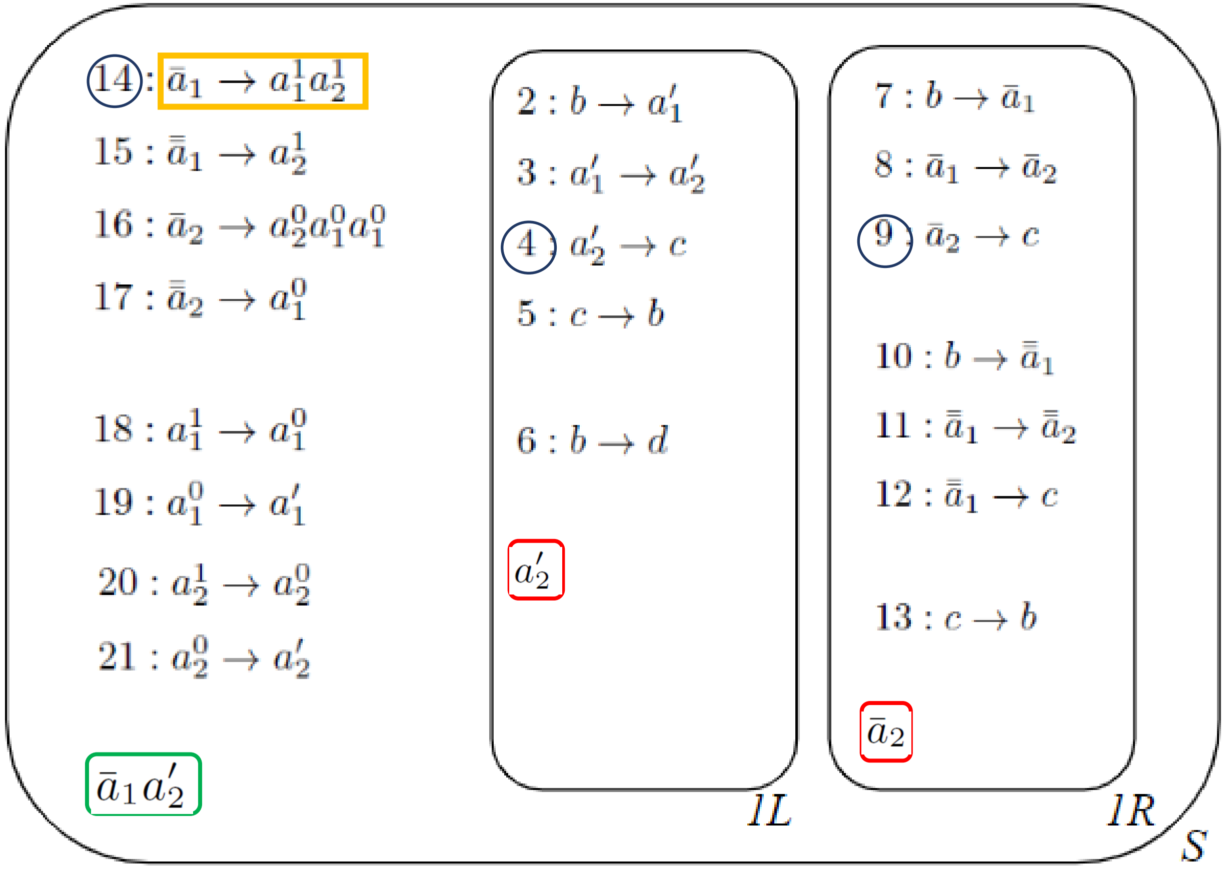

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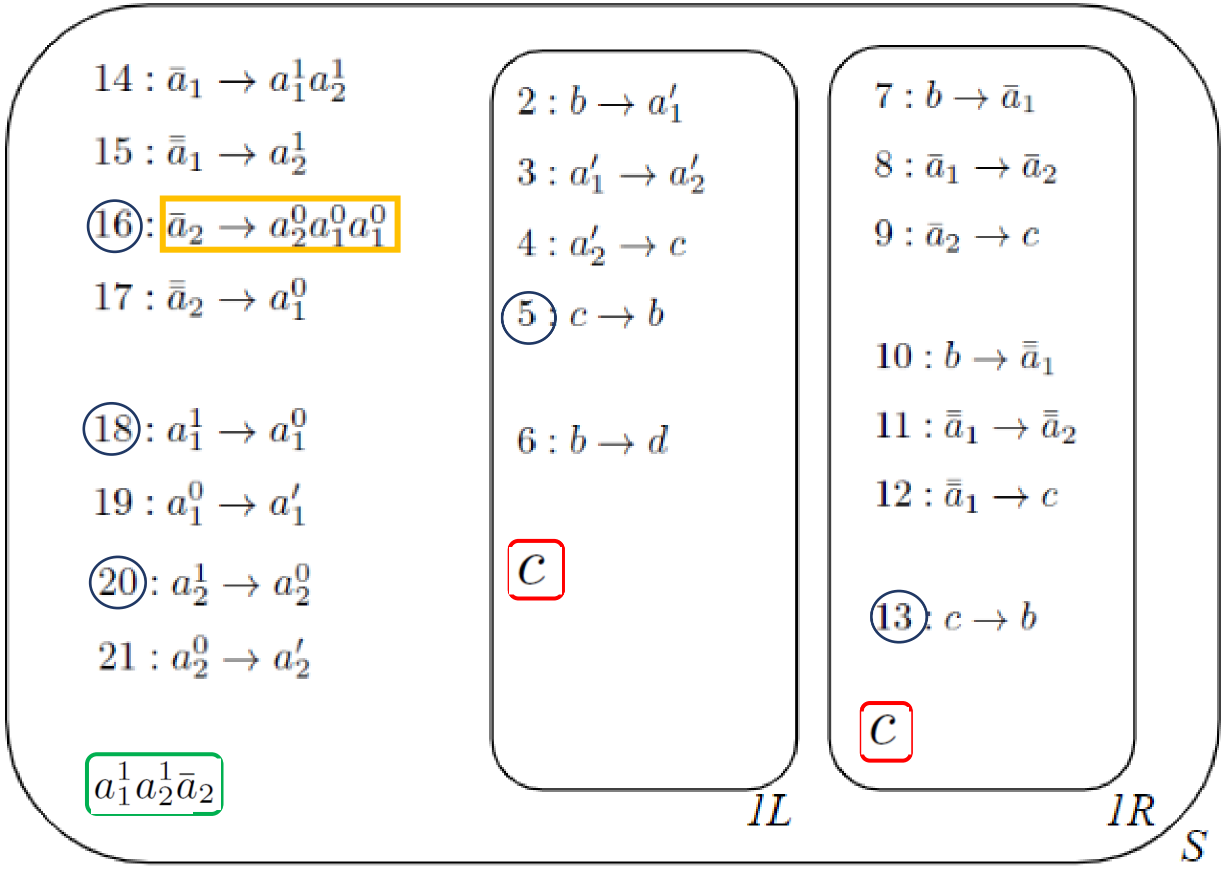

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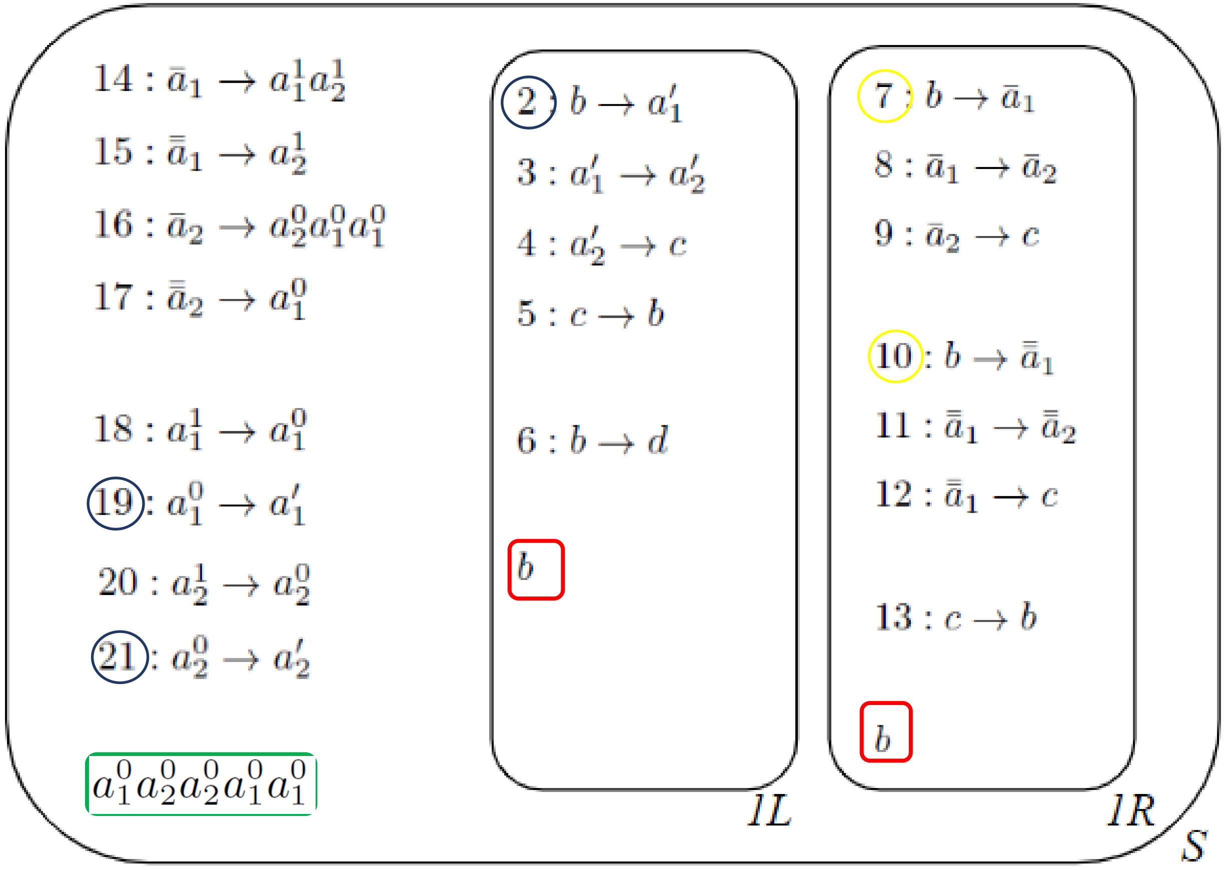

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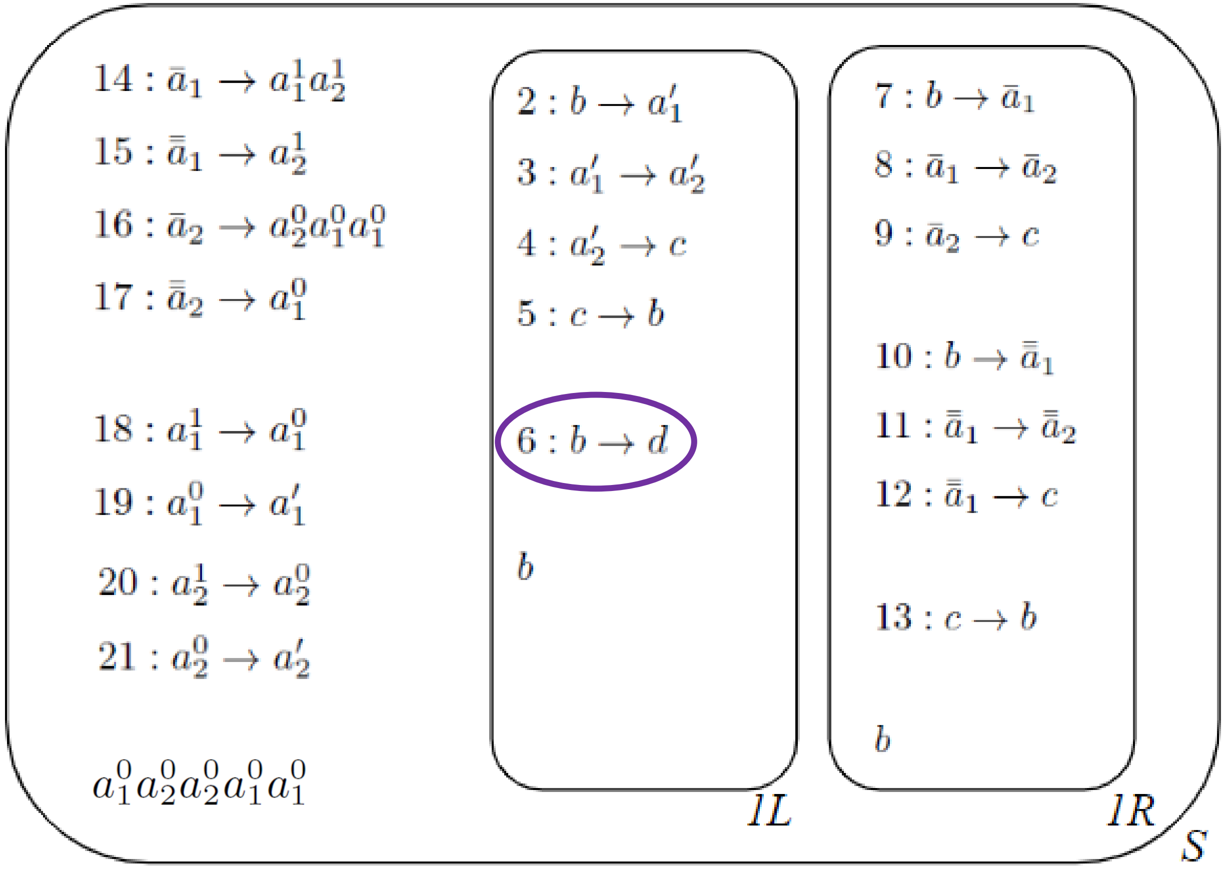

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Systems with finite sets of instances of dynamic rules

- **Non-cooperative** rules → Left-membranes have **finitely many** possible **membrane contents** in any computation

→left-membranes are always "**finitely representable**"

- What about "finitely representable" **right-membranes**?

Finite representabilty

$$\Pi = (O, T, \mu, w_s, \langle w_{1L}, w_{1R} \rangle, \ldots, \langle w_{nL}, w_{nR} \rangle, h_o)$$

- If w_h is the contents of **region *h*** after the ***j*-th step** of a computation, and w'_h can be obtained from w_h in the **next** computational **step:**

$$w'_h \in \sigma_{j,h}(w_h)$$

- and $\sigma^0_{j,h}(w_h) = w_h$,

  $\sigma^{k+1}_{j,h} = \sigma_{j+k,h}(\sigma^k_{j,h}(w_h))$ (set of contents obtainable in k+1 steps)

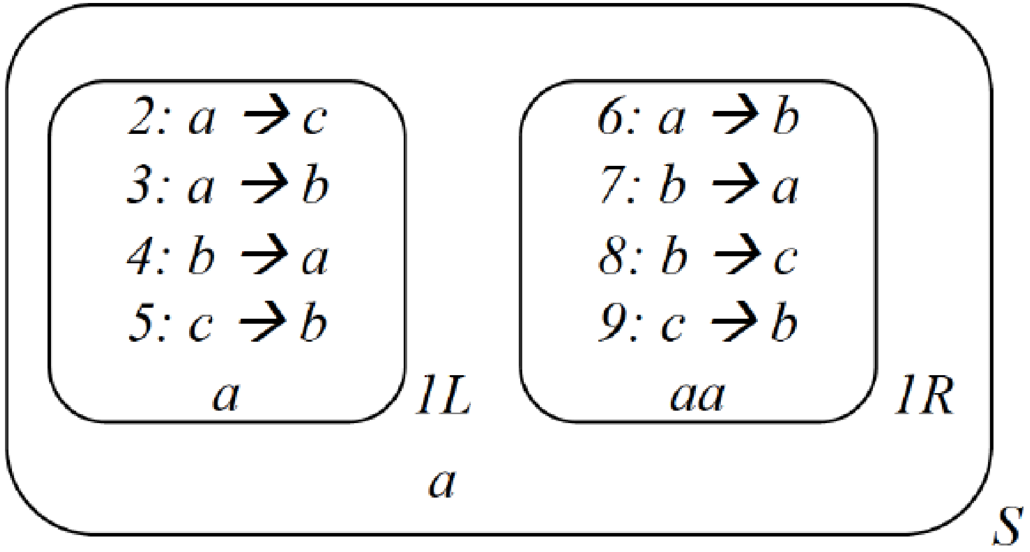
- $\sigma^*_{j,h} = \bigcup_{k \geq 0} \sigma^k_{j,h}(w_h)$

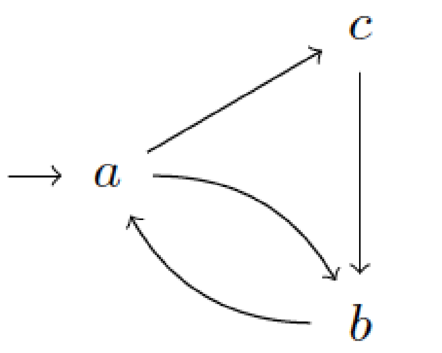
Finite representability

Region ***h*** **is FIN-representable** if the **set of successor multisets** of the initial contents w_h of region *h* is **finite**.

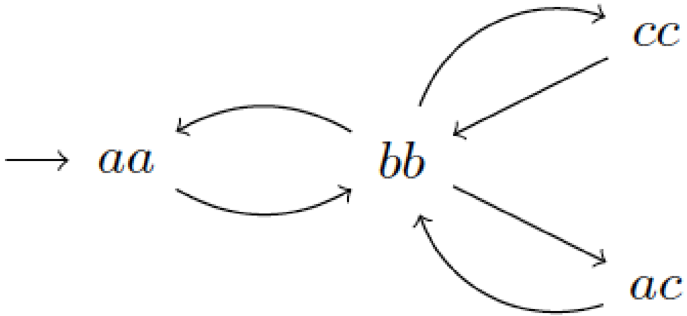
→ if $\sigma^*_{0,h}(w_h)$ is finite

FIN-representability, an example





$$\sigma^*_{0,1L}(a) = \{a, b, c\}$$



$$\sigma^*_{0,1R}(aa) = \{aa, bb, cc, ac\}$$

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Region *S* is not FIN-representable

A characterization of *PsET0L*

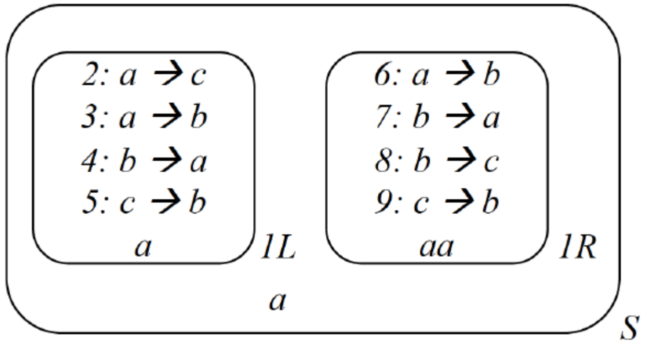
Theorem: $\mathcal{L}(NOP(polym\ \ , ncoo, fin)) \subseteq PsET0L.$

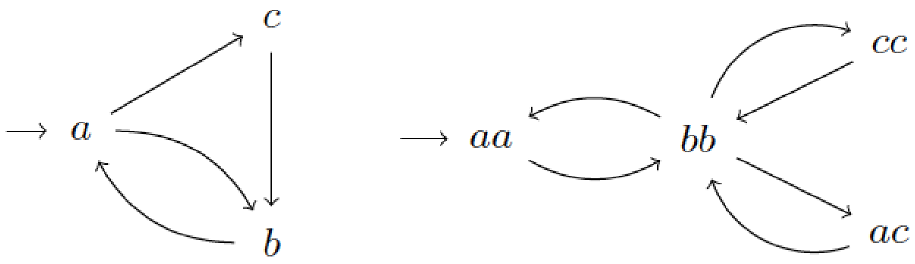
Corollary: $\mathcal{L}(NOP(polym\ \ , ncoo, fin)) = PsET0L.$

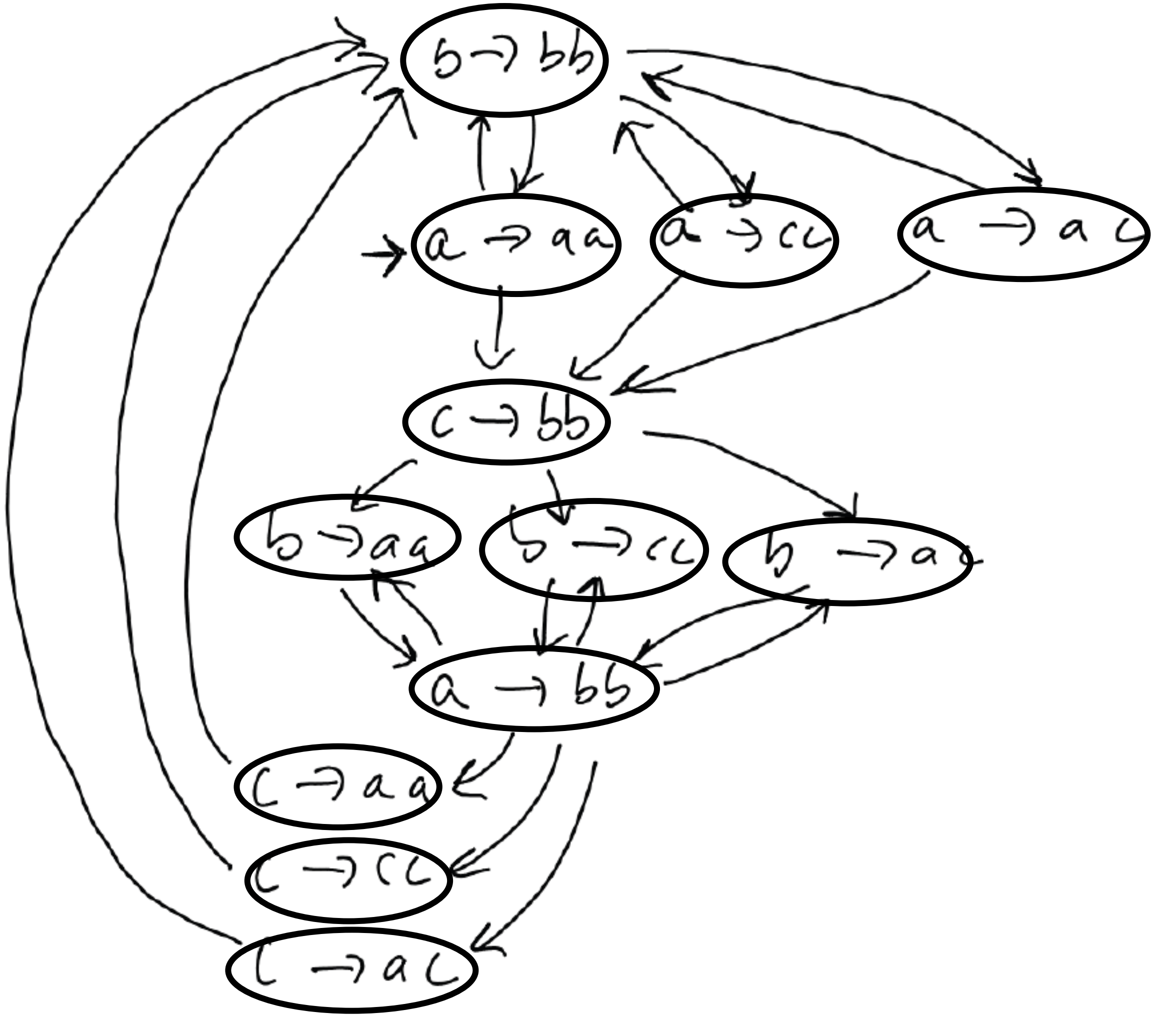
Why does the corollary hold?

The proof idea

We can construct the **finite set** of instances of **rule 1**:







The proof idea

The **construction** of the *ET0L* **tables**:

- initial string: $d_{a \to aa}\ a$
- the table:

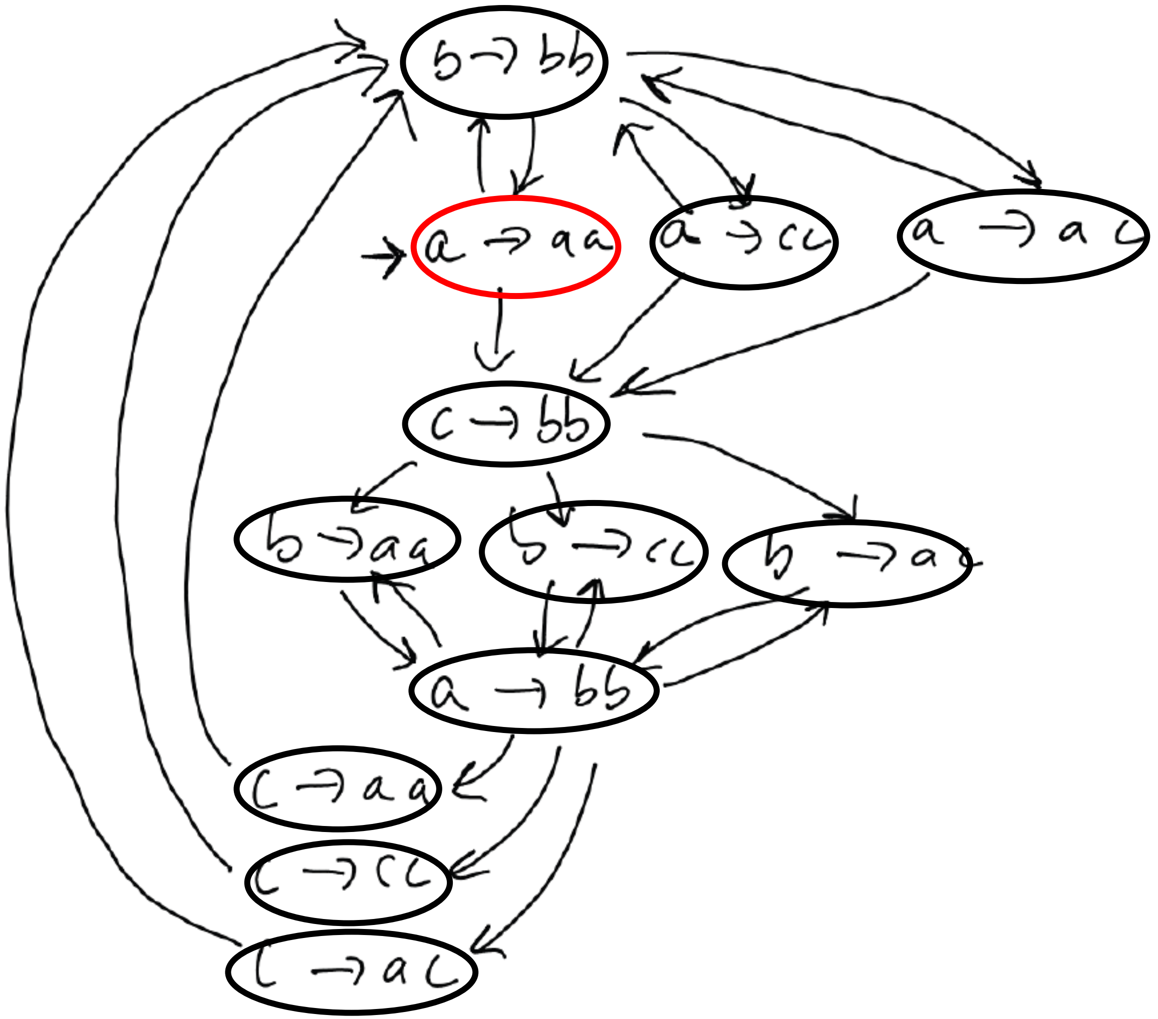
  $a \to aa$

  $d_{a \to aa} \to d_{b \to bb}$

  $d_{a \to aa} \to d_{c \to bb}$

  $d_{x \to yz} \to F$

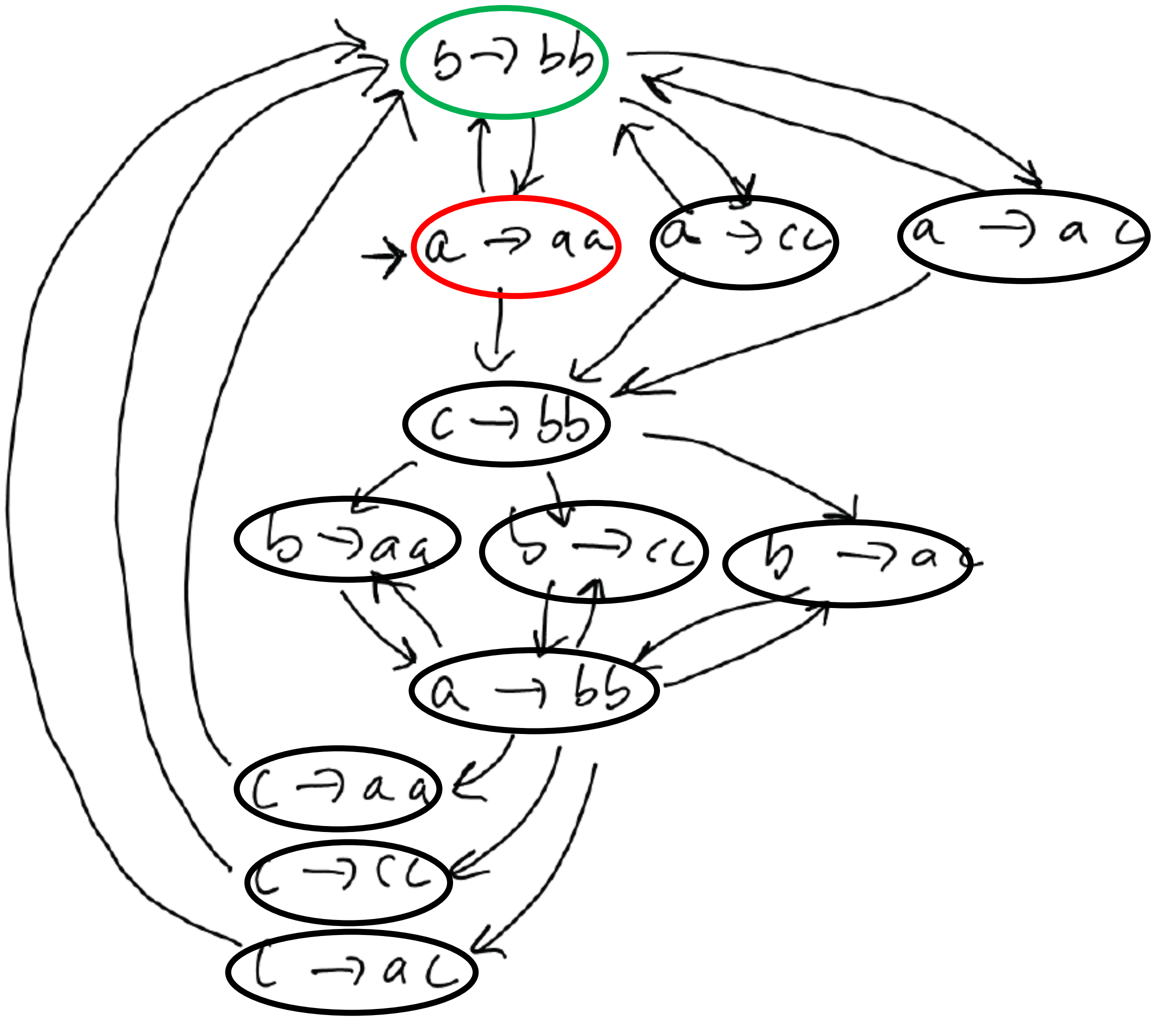
  $F \to F$



The proof idea

The **construction** of the *ET0L* **tables**:

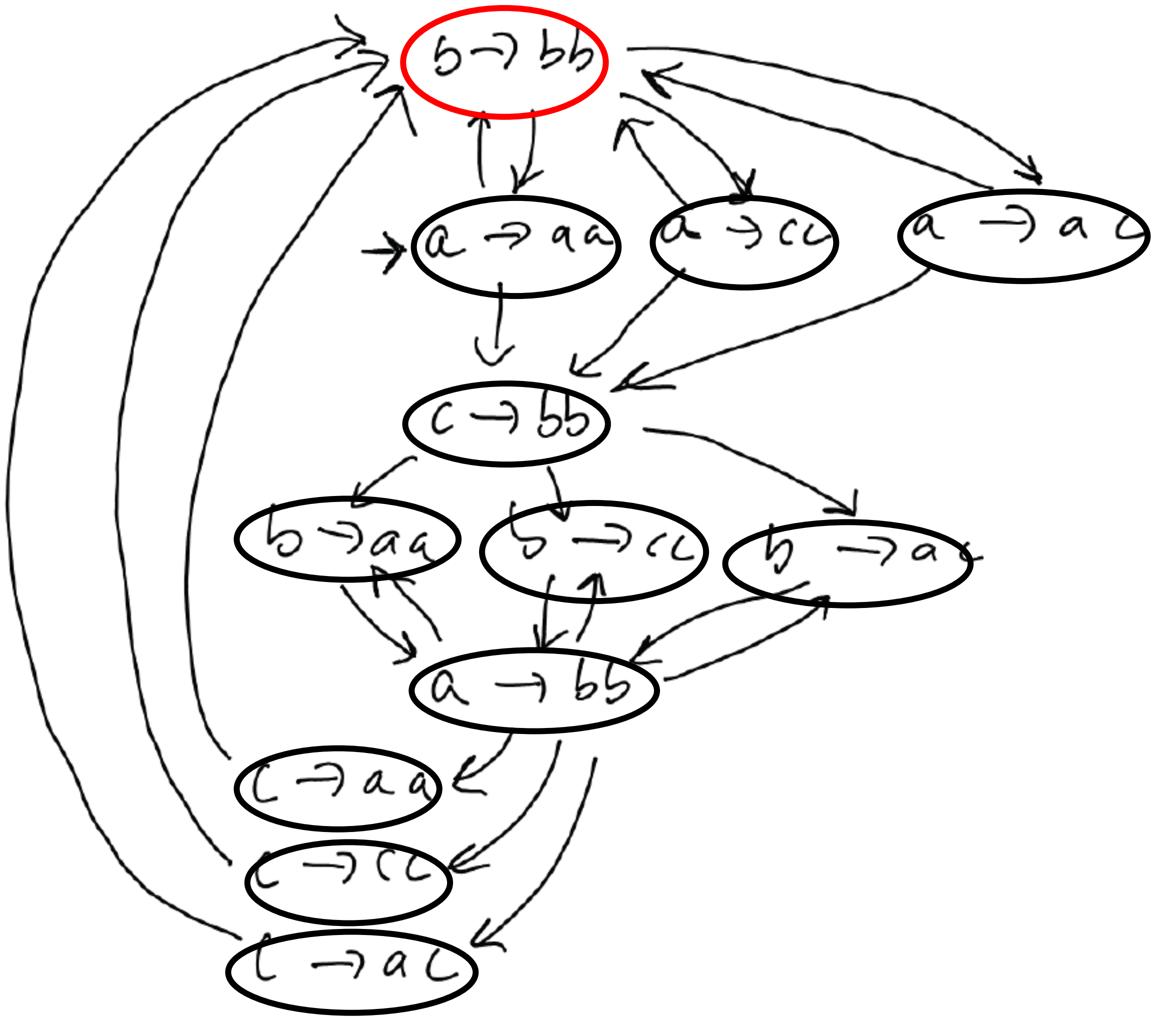
- after the *1st* step: $d_{b\to bb}\ aa$
- the table:
  $a \to aa$
  $d_{a\to aa} \to d_{b\to bb}$
  $d_{a\to aa} \to d_{c\to bb}$
  $d_{x\to yz} \to F$
  $F \to F$



The proof idea

The **construction** of the *ET0L* **tables**:

- after the 1st step: $d_{b \to bb}\ aa$
- the table:
  $b \to bb$
  $d_{b \to bb} \to d_{a \to aa}$
  $d_{b \to bb} \to d_{a \to cc}$
  $d_{b \to bb} \to d_{a \to ac}$
  $d_{x \to yz} \to F$
  $F \to F$



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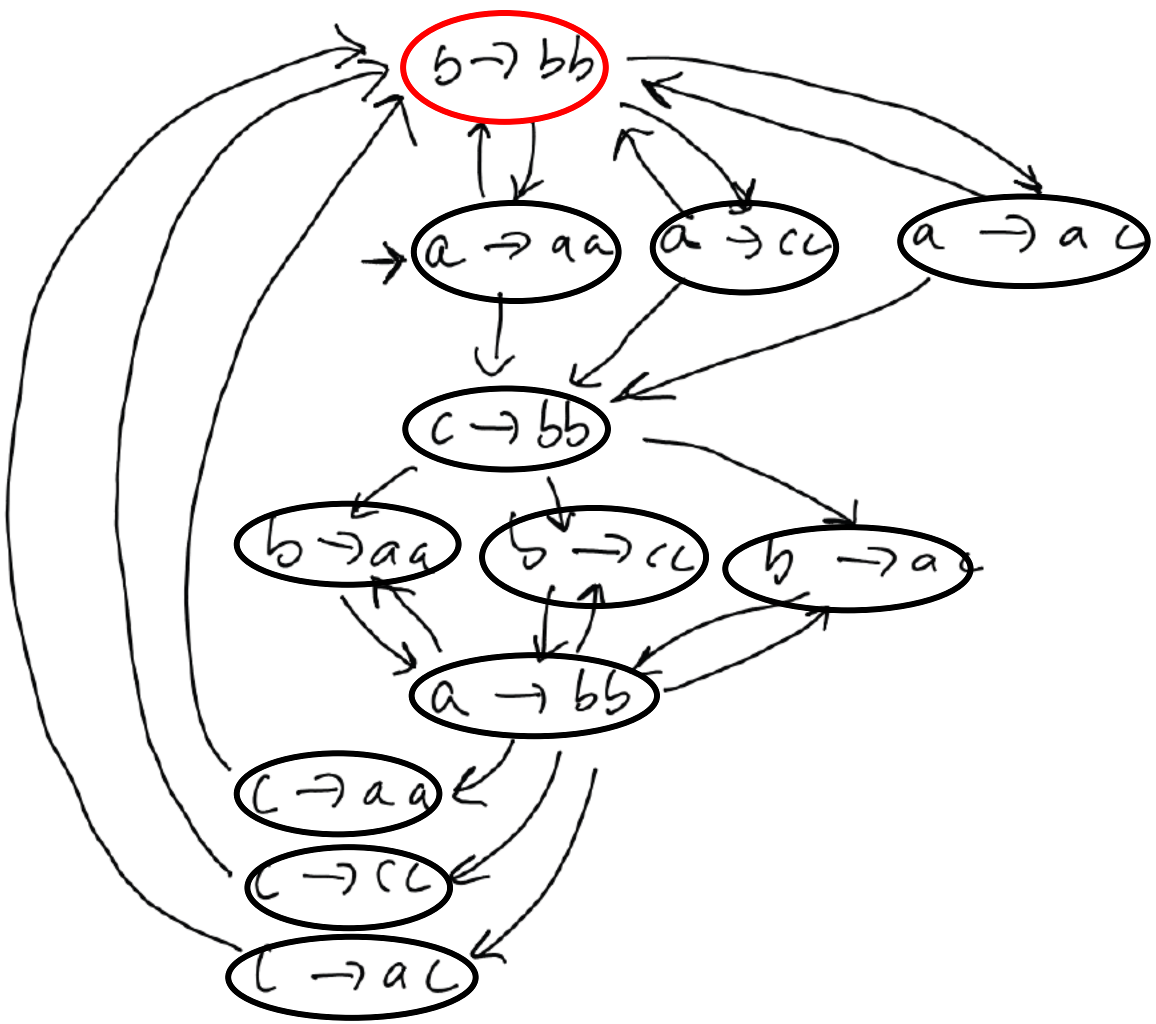
The proof idea

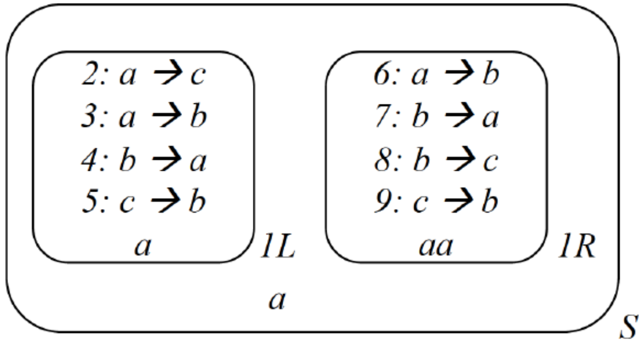
The **construction** of the *ET0L* **tables**:

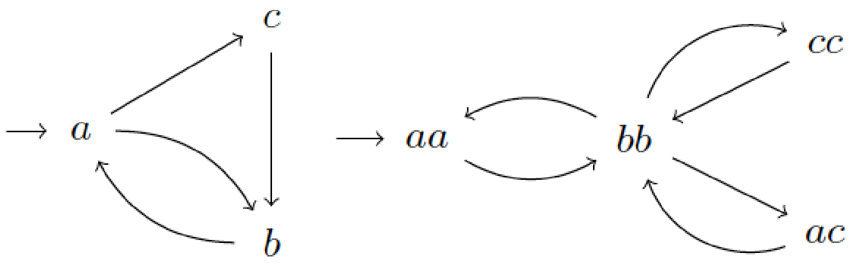
- after the 1st step: $d_{b \to bb}\ aa$
- the table:
  $b \to bb$
  $d_{b \to bb} \to d_{a \to aa}$
  $d_{b \to bb} \to d_{a \to cc}$
  $d_{b \to bb} \to d_{a \to ac}$
  $d_{x \to yz} \to F$
  $F \to F$ and so on…

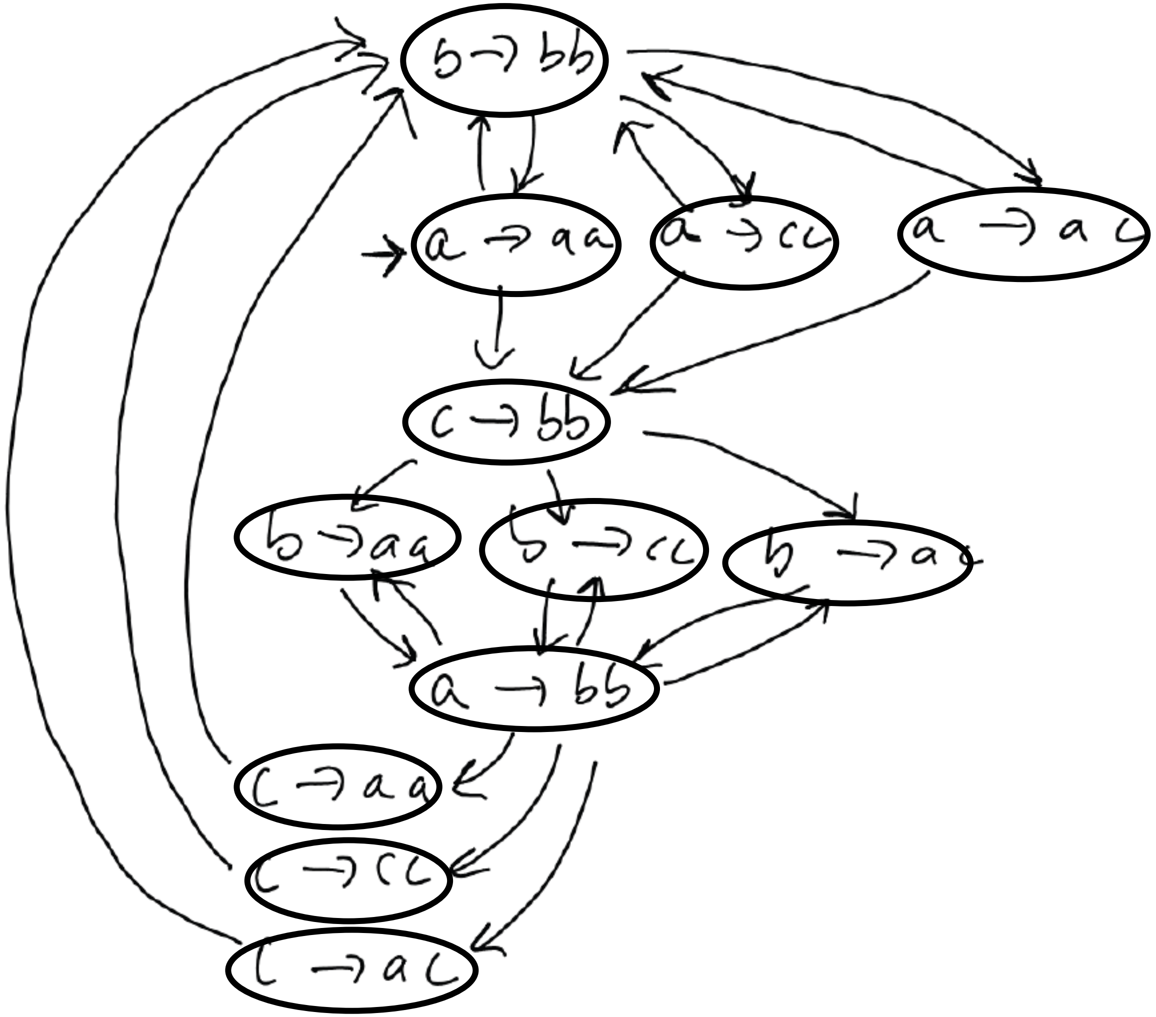


If we have two dynamic rules…

…we can **construct** the finite set of instances of **rule pairs**

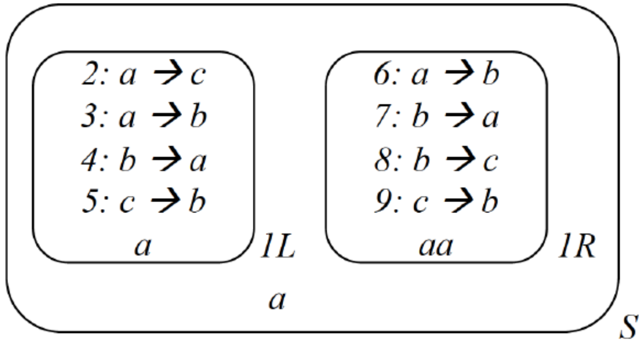


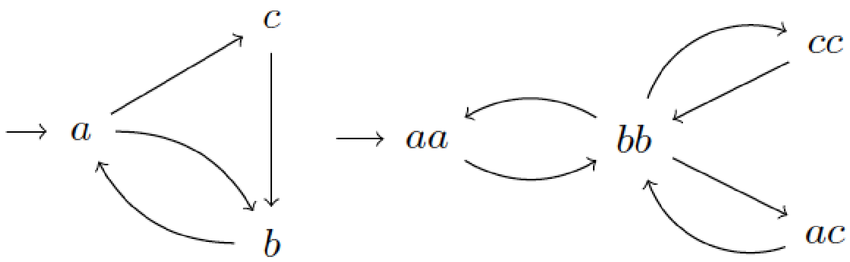


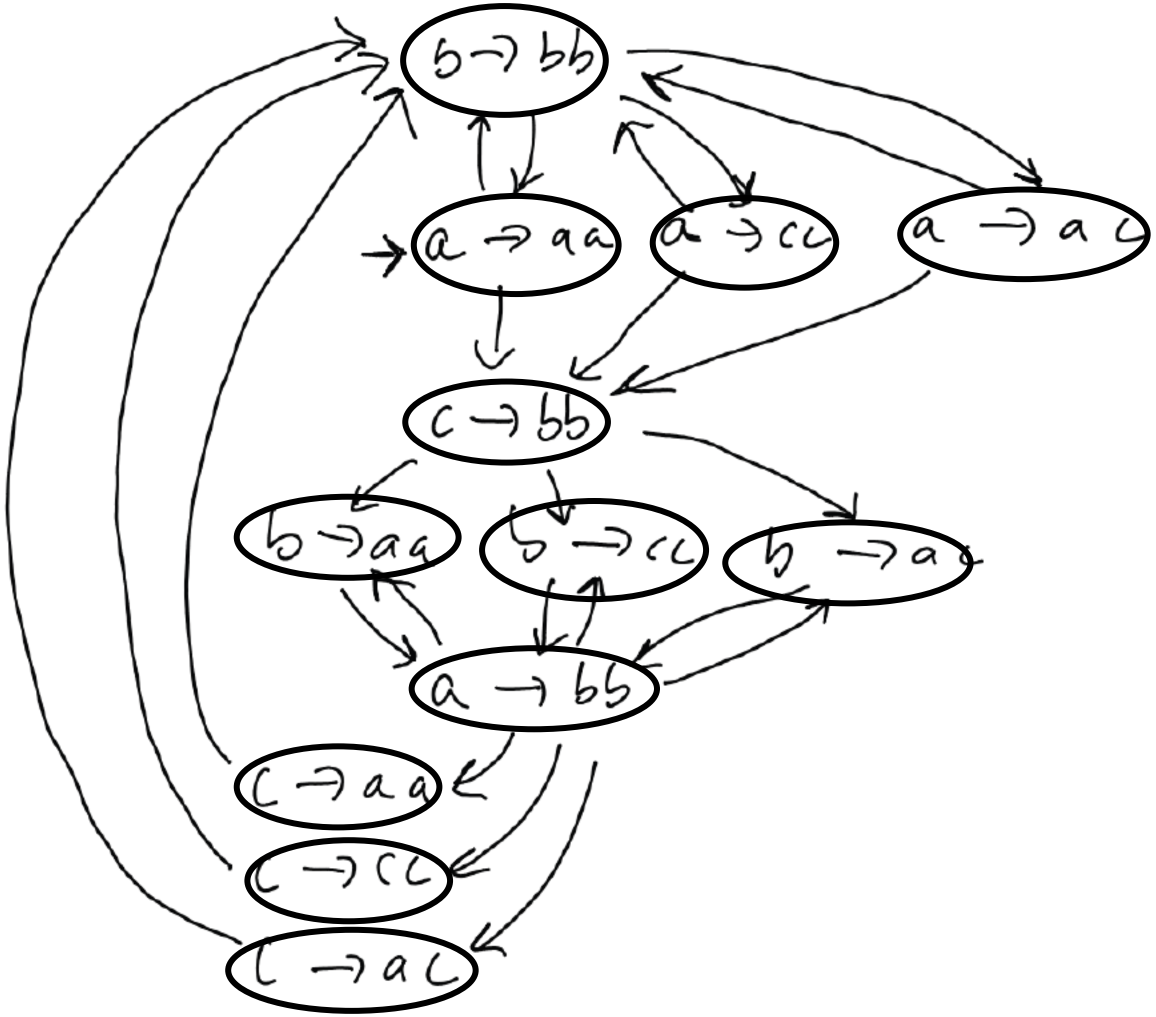


If we have several dynamic rules…

…we can construct the finite set of instances of **groups of rules that can be applied simultaneously**







Outline

- Polymorphic P systems
  - The idea and the model
  - A few basic properties
- Polymorphic P systems with non-cooperative rules and no ingredients

- Non-cooperative polymorphic P systems with limited depth
- Non-cooperative polymorphic P systems with "finitely representable" regions

  → a characterization of *PsET0L*

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Thank you.