cP systems and QUBO

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Outline

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cP systems

Introduction

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cP system = P system with compound terms[1]

[1] Radu Nicolescu and Alec Henderson. "An Introduction to cP Systems". In: *Enjoying Natural Computing: Essays Dedicated to Mario de Jesús Pérez-Jiménez on the Occasion of His 70th Birthday*. Ed. by Carmen Graciani et al. Cham: Springer International Publishing, 2018, pp. 204–227. isbn: 978-3-030-00265-7. doi: 10.1007/978-3-030-00265-7_17.

cP systems

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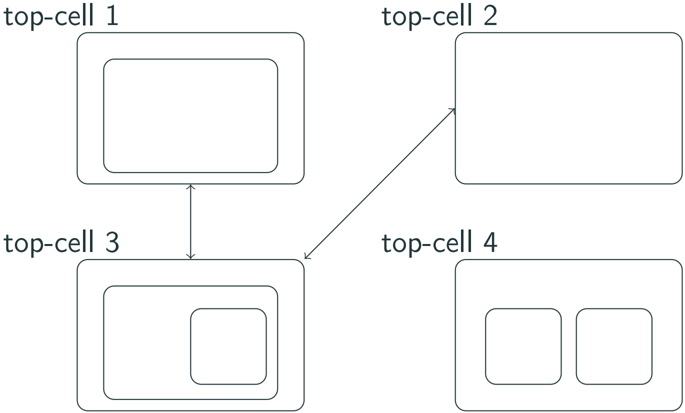
Formally, a cP system is a construct

$$\Pi = (T, A, O, C, R, S, \overline{s}), \text{ where}$$

T is the set of top-level cells at the start of the evolution of the system; A is the alphabet of the system; O is the set of multisets of initial objects in the top-level cells; C is the set of sets of channel endpoint labels for inter-top-level cell communication that can be found in each top-level cell; R is the set of rule-sets for each top-level cell; S is the set of possible states of the top-level cells; and $\overline{s} \in S$ is the starting state of every top-level cell in the system.

cP systems



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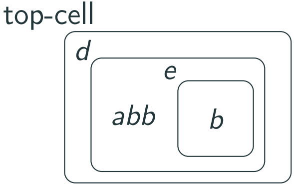

cP systems

cell system	corresponding terms and compound terms
top-cell $\boxed{abb}$	ab^2

cP systems

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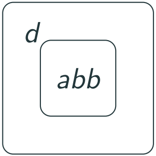
cell system	corresponding terms and compound terms
top-cell 	$d(ab^2)$

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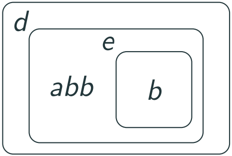
cell system	corresponding terms and compound terms
top-cell 	$d(ab^2\ e(b))$

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Rule

$$\mathit{current\text{-}state}\ \mathit{lhs} \longrightarrow\ \mathit{target\text{-}state}\ \mathit{rhs}$$

States can be omitted

lhs and *rhs* contain terms, compound terms and **variables**

Example

$lhs = +(aX)Y^2$

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Rule

$$S \ + (aX)Y^2 \rightarrow \ S' + (XY)$$

Example

top-level cell: $+(a^2c)b^2$ — there is only one matching $X = ac, \ Y = b$.

$$S \ + (aac)b^2 \rightarrow \ S' + (acb)$$

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Rule

$$S \ +(XY) \rightarrow \ S' + (X)Y^2$$

Example

top-level cell: $+(ab)$ — there are four sets of matching
$X = a, \ Y = b; \ X = b, \ Y = a; \ X = \lambda, \ Y = ab; \ X = ab, \ Y = \lambda.$

$$\begin{aligned}
S \ +(ab) &\rightarrow \ S' + (a)b^2 \\
S \ +(ba) &\rightarrow \ S' + (b)a^2 \\
S \ +(ab) &\rightarrow \ S' + ()a^2 b^2 \\
S \ +(ab) &\rightarrow \ S' + (ab)
\end{aligned}$$

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Modes

min mode – one of matching rules is executed

max mode – ALL the rules can be applied

Rule

$$S \ + (XY) \rightarrow \ S' + (X)Y^2$$

$$S \ + (ab) \rightarrow \ S' + (a)b^2$$
$$S \ + (ba) \rightarrow \ S' + (b)a^2$$
$$S \ + (ab) \rightarrow \ S' + ()a^2b^2$$
$$S \ + (ab) \rightarrow \ S' + (ab)$$

Rules

Rule

$$S \; + (\_) \longrightarrow_{max} \; S'$$

_ – anonymous variable

Rules, inhibitors, promoters

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Rule

$$S\ +() \longrightarrow\ S' + (X) \mid -(X1)$$
$$S\ +(X) + (Y) \longrightarrow\ S'c(XY) \mid \neg d(\lambda)$$
$$S\ +(X) + (Y) \longrightarrow\ S'c(XY) \mid \neg(X = Y)$$

cP systems and integers

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For every integer a there is

$$i(x,y),\ \text{where } x,y \in \mathbb{N}_0 \iff a = x - y$$

For example:

$i(1,4) \iff -3 = 1-4$
$i(5,8) \iff -3 = 5-8$
$i(8,11) \iff -3 = 8-11$
$i(0,3) \iff -3 = 0-3$

Every representation $i(x,y)$ can be converted into canonical form:

$$i(x,y) \sim \begin{cases} i(x',0) & \text{for } x \geq y \quad \text{where } x' = x - y \\ i(0,y') & \text{for } x < y \quad \text{where } y' = y - x \end{cases}$$

cP systems and integers

Addition

$$i(x,y)+i(x',y')=i(x+x',y+y')$$

Subtraction

$$i(x,y)-i(x',y')=i(x+y',y+x')$$

Multiplication

$$i(x,y)\cdot i(x',y')=i(x\cdot x'+x'\cdot y',y\cdot x'+x\cdot y')$$

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$$
\begin{aligned}
i(3,2) &\rightsquigarrow i\,(+(111)\ -(11)) = i\,(+(3)\ -(2)) \\
i(1,4) &\rightsquigarrow i\,(+(1)\ -(1111)) = i\,(+(1)\ -(4)) \\
i(2,0) &\rightsquigarrow i\,(+(11)\ -()) = i\,(+(2)\ -()) \\
i(0,0) &\rightsquigarrow i\,(+()\ -())
\end{aligned}
$$

cP systems and integers

Addition

$$\longrightarrow_{\min} k\,(+(AC)\ -(BD))\ \mid\ i\,(+(A)\ -(B))\ \ j\,(+(C)\ -(D))$$

Subtraction

$$\longrightarrow_{\min} k\,(+(AD)\ -(BC))\ \mid\ i\,(+(A)\ -(B))\ \ j\,(+(C)\ -(D))$$

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QUBO problem

Quadratic Unconstrained Binary Optimisation is an NP-hard mathematical optimization problem.

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QUBO problem

Integer version of the problem of minimizing a quadratic objective function

$$x^* = \min_{\vec{x}} \vec{x}^T Q \vec{x}$$

where:

- $\vec{x}$ is a n-vector of binary (Boolean) variables $x_i \in \{0;1\}, 0 \le i \le n-1$
- $n \in \mathbb{N}_0$ - the number of variables in $\vec{x}$
- $i, j \in \mathbb{N}_0$
- Q is an upper-triangular $n \times n$ matrix where $q_{i,j} \in \mathbb{Z}, 0 \le i \le j \le n-1$ are possibly non-zero coefficients

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QUBO problem

Formally, QUBO problems are of the form:

$$x^* = \min_{\vec{x}} \sum_{i \le j} x_i q_{i,j} x_j, \quad \text{where } x_i, x_j \in \{0,1\}$$

$5x_0^2 - 7x_1^2 + x_2^2 - 2x_0x_1 + x_1x_2 \quad ? \quad (0,1,1)$

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We developed a cP system that finds minimal value of a QUBO in three phases of computation.

1. In the first phase, all possible values assignment is generated.
2. The second phase is devoted to generating of all polynomials.
3. In the third phase, related coefficients are added together to evaluate potential solutions for the assignments produced from phases 1 and 2.

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Input:

- For every variable x_i storing value $y_i \in \{0,1\}$ there is complex object
$$a\big(in(i)val(y_i')\big) \text{ where } y_i' \in \{\lambda, 1\}.$$
- For every coefficient $q_{i,j}$ there is complex object
$$q\,(in1(i)in2(j)val(+(x)\ -(y))) \text{ where } q_{i,j} = x - y.$$
- Two counters (counter-like objects): $C_1(\lambda), C_2(n)$.
- Empty list of values of variables: $l(C_1(\lambda))$ with counter $C_1(\lambda)$ inside.

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$$\boxed{S_1 \quad C_2(1X) \longrightarrow_{\min} S_2 \quad v(\lambda)v(1)C_2(X) \qquad (1)}$$

Skin membrane contains complex object $C_2(n)$ – $n = 1^n$

Unified rule:

$$S_1 \quad C_2(11^{n-1}) \longrightarrow_{\min} S_2 \quad v(\lambda)v(1)C_2(1^{n-1})$$

By the execution of the rule (1), two complex objects - $v(\lambda)$ and $v(1)$ are generated and the number of 1s inside $C_2()$ is decreased by one.

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$$
\begin{array}{llll}
S_2 & \longrightarrow_{\max} & S_1 & l(a(in(X)val(Y))\,C_1(X1)Z) \quad | \; l(C_1(X)Z) \\
 & & & \qquad\qquad\qquad\qquad\qquad\quad | \; v(Y)
\end{array}
$$

1. round –

$$
\begin{array}{llll}
S_2 & \longrightarrow_{\max} & S_1 & l(a(in(\lambda)val(\lambda))\,C_1(1)\lambda) \quad | \; l(C_1(\lambda)\lambda) \\
 & & & \qquad\qquad\qquad\qquad\qquad\quad | \; v(\lambda) \\
S_2 & \longrightarrow_{\max} & S_1 & l(a(in(\lambda)val(1))\,C_1(1)Z) \quad | \; l(C_1(\lambda)\lambda) \\
 & & & \qquad\qquad\qquad\qquad\qquad\quad | \; v(1)
\end{array}
$$

1. $x_0 = 0$ 2. $x_0 = 1$

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$$
\begin{aligned}
S_2 \quad \longrightarrow_{\max} \quad S_1 \quad & l(a(in(X)val(Y))\,C_1(X1)Z) \quad \mid l(C_1(X)Z) \\
& \mid v(Y)
\end{aligned}
$$

2. round – for $a(in(\lambda)val(\lambda))$ there are two rules

$$
\begin{aligned}
S_2 \quad \longrightarrow_{\max} \quad S_1 \quad & l(a(in(1)val(\lambda))\,C_1(1)a(in(\lambda)val(\lambda))) \\
& \mid l(C_1(\lambda)a(in(\lambda)val(\lambda))) \\
& \mid v(\lambda) \\
S_2 \quad \longrightarrow_{\max} \quad S_1 \quad & l(a(in(1)val(1))\,C_1(1)a(in(\lambda)val(\lambda))) \\
& \mid l(C_1(\lambda)a(in(\lambda)val(\lambda))) \\
& \mid v(1)
\end{aligned}
$$

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$$\boxed{\begin{array}{llll} S_2 & l(\_) & \longrightarrow_{\max} & S_1 \\ S_2 & v(\_) & \longrightarrow_{\max} & S_1 \end{array}}$$

In the same step all $l()$ and $v()$ that serve as promoters are erased.

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The second phase

The idea of the second phase is to generate objects $p()$, which will contain representatives of quadratic elements that will be multiplied by the coefficients of one (say the i-th) row of the matrix Q. However, if the value of the variable x_i is zero, the generation of the row is omitted since its value will be zero.

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$\vec{x} = (1,1,1)$ — $m()$ will contain the following objects:

$$
\begin{array}{lllll}
p(w(\lambda) & a(in(\lambda)val(1)) & a(in(1)val(1)) & a(in(11)val(1)) &) \\
p(w(1) & & a(in(1)val(1)) & a(in(11)val(1)) &) \\
p(w(11) & & & a(in(11)val(1)) &)
\end{array}
$$

$$
\begin{array}{llll}
x_0 & x_0 & x_1 & x_2 \\
x_1 & & x_1 & x_2 \\
x_2 & & & x_2
\end{array}
$$

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$\vec{x} = (1,0,1)$ — $m()$ will contain the following objects:

$$
\begin{array}{lllll}
p(w(\lambda) & a(in(\lambda)val(1)) & a(in(1)val(\lambda)) & a(in(11)val(1)) &) \\
p(w(1) & & & &) \\
p(w(11) & & & a(in(11)val(1)) &)
\end{array}
$$

$$
\begin{array}{lllll}
x_0 & & x_0 & x_1 & x_2 \\
x_1 = 0 & & & x_1 & x_2 \\
x_2 & & & & x_2
\end{array}
$$

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The third phase

In the third phase, for each combination of non-zero values of $x_i x_j$, we will add the value of the coefficient q_{ij} to the result in the object $I()$. After that we convert all values to canonical form. If there is at least one negative number we search for the maximum of negative numbers, if there is no negative number we search for the minimum of positive numbers (and zero).

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adding the number q_{XY} if both x_X and x_Y are 1

$$
\boxed{
\begin{array}{l}
S_4 \quad l\,(r\,(+(U')\ -(V'))\,p\,(w(X)a\,(in(Y)val(1))\,Z)\,Z'\,C_1(X)) \\
\qquad \longrightarrow_{\max} \quad S_4 \quad l\,(r\,(+(UU')\ -(VV'))\,p\,(w(X)Z)\,Z'\,C_1(X)) \\
\qquad\qquad\qquad\qquad \mid\ q\,(in1(X)in2(Y)val(+(U)\ -(V)))
\end{array}
}
$$

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$$
\begin{aligned}
S_4 \quad & l(p(w(X)a(in(Y)val())Z)Z'C_1(X)) \\
& \longrightarrow_{\max} \quad S_4 \quad l(p(w(X)Z)Z'C_1(X)) \\
& \qquad\qquad\qquad\qquad \mid q(in1(X)in2(Y)val(Z''))
\end{aligned}
$$

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Normalisation:

$$S_5 \quad l(r(+(XY)\ -(Y))\_) \quad \longrightarrow_{\max} \quad S_6 \quad l(r(+(X)\ -(\lambda))\_)$$

$$S_5 \quad l(r(+(X)\ -(XY))\_) \quad \longrightarrow_{\max} \quad S_6 \quad l(r(+(\lambda)\ -(Y))\_)$$

cP systems and QUBO

Finding maximum of negative part of $r()$

$$
\begin{aligned}
S_7 \quad \longrightarrow_{\min} \quad S_F \quad & res(val(+(\lambda)\ -(X))Z) \\
& \qquad \mid l(r(+(\lambda)\ -(X))Z) \\
& \qquad \neg l(r(+(\lambda)\ -(X1Y))\_)
\end{aligned}
$$

cP systems and QUBO

To find a minimum of positive and zero values we need to add one to the content of $+()$ so the value of each $r()$ is at least one. Then we find a minimum of positive parts of $r()$s.

$$S_8 \quad l(r(+(X)\ -(\lambda))Z) \quad \longrightarrow_{\max} \quad S_9 \quad l(r(+(X1)\ -(\lambda))Z)$$

$$\begin{aligned} S_9 \quad \longrightarrow_{\min} \quad S_F \quad & res(val(+(X)\ -(\lambda))Z) \\ & \mid l(r(+(1X)\ -(\lambda))Z) \\ & \neg(X = YW)\ l(r(+(Y)\ -(\lambda))\_) \end{aligned}$$

Questions? Comments?

Thank you for your attention!